When to use semi-classical GR instead of QFT to probe cosmological correlation functions: a quantitative view

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Introduction

The universe's inhomogeneities might have been **quantum fluctuations** during inflation. If so,

- Can we tell **when it stopped** up to a precision criterion?
- Can we characterise the **signatures of classical vs quantum** inhomogeneities? [1]

These are particularly important question given that

- **Classical simulations of inflation** have become a norm [2,3,4,…]
- The quantumness of initial conditions has not been observed yet [5].

From quantum to classical inflation

- The quantification of perturbations in the inflaton and geometry via Bunch-Davies vacuum is standard

$$\mathcal{R}_k(\eta) = -\frac{H}{M_{Pl}}\sqrt{\frac{\pi}{8\varepsilon_1 k^3}}(-k\eta)^{3/2}H_\nu^{(1)}[-k\eta] \underset{\varepsilon_{1,2}\ll 1}{\simeq} \frac{iH}{M_{Pl}}\frac{1}{\sqrt{4\varepsilon_1 k^3}}(1+ik\eta)e^{-ik\eta}$$

- The **commutators and anti-commutators** are related

$$\begin{cases} \left\langle \left\{\hat{\mathcal{R}}(\mathbf{x},t),\hat{\mathcal{R}}(\mathbf{y},t')\right\}\right\rangle = 2\int d^3k e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{y})}\,\mathrm{Re}\left[\mathcal{R}_k(t)\mathcal{R}_k^*(t')\right] := 2\int d^3k e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{y})}F_k(t,t') \\ \left\langle \left[\hat{\mathcal{R}}(\mathbf{x},t),\hat{\mathcal{R}}(\mathbf{y},t')\right]\right\rangle = 2\int d^3k e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{y})}\,\mathrm{Im}\left[\mathcal{R}_k(t)\mathcal{R}_k^*(t')\right] := \int d^3k e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{y})}G_k(t,t') \end{cases}$$

- **SuperHubble limit** $\quad F_k(t_f,t) \underset{k\ll aH(t)}{\gg} G_k(t_f,t)$

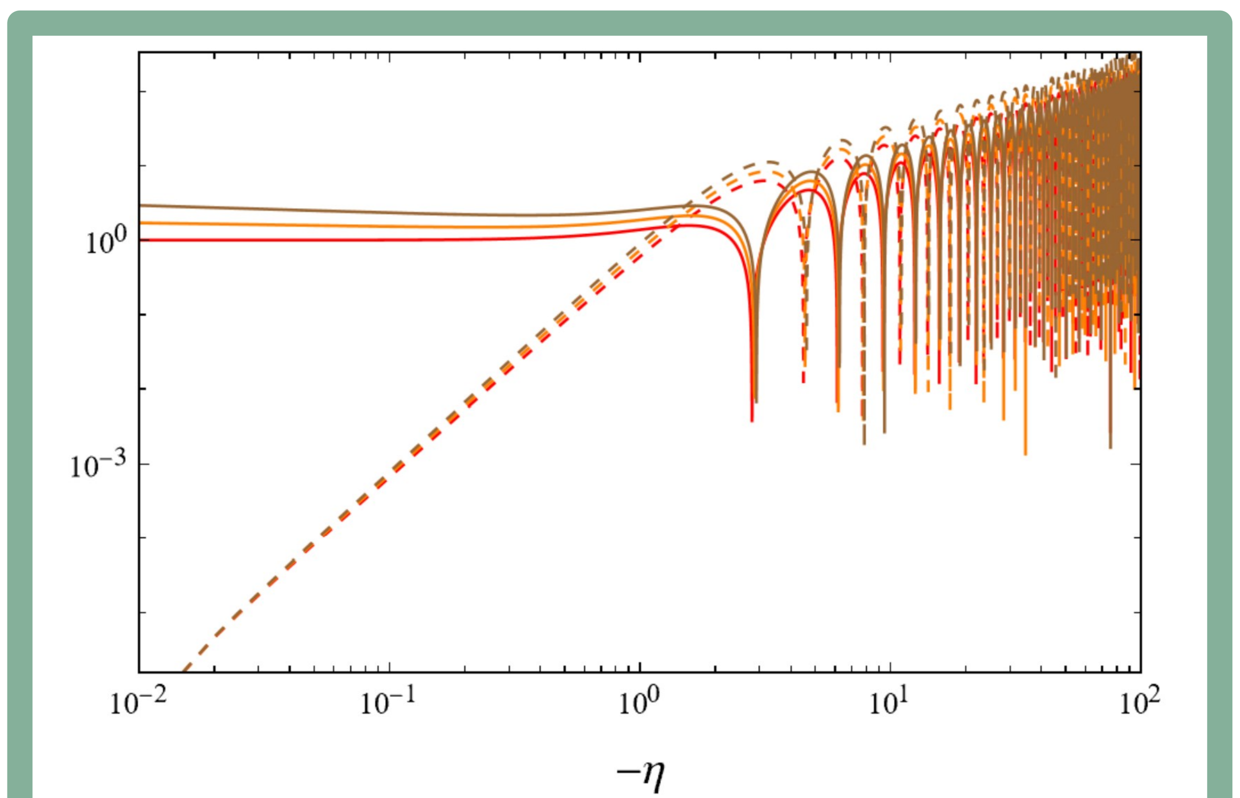


Figure 1: Absolute ($k=1$) F (solid lines) and G (dashed lines) in quasi-dS ($\nu = 3/2 + 0$ (red) , +0.05 (orange) , +0.1 (brown)) for η up to $\eta_f = -0.01$ and, up to constants. Oscillations amplify when $\eta = -\infty$.

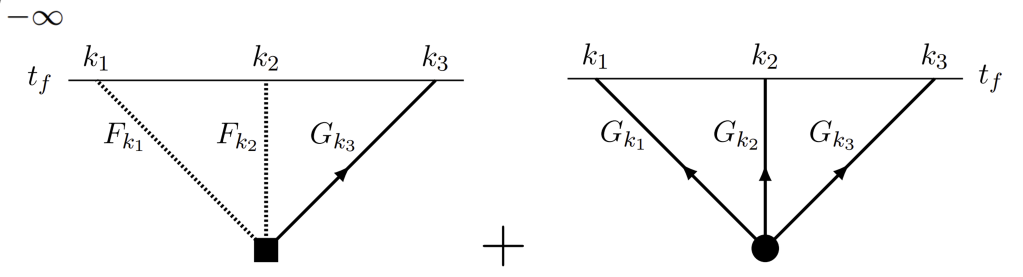
- Interactions are much more complicated (integrals of products of propagators) and very much quantum-imprinted. We focus on the **bispectrum.**

Semi-classical correlation function

- The bispectrum (3pt correlation function) is the **first higher correlator with interactive contribution** (leading order), still being measured.
- The **Keldysh** diagrammatics [6] dissociate on-shell from off-shell dynamics in interacting theories (e.g. $\lambda\phi^3$ here), here when propagating **from past infinity**

$$\langle \hat{\mathcal{R}}_{\mathbf{k}_1}\hat{\mathcal{R}}_{\mathbf{k}_2}\hat{\mathcal{R}}_{\mathbf{k}_3}|_{-\infty}^{t_f}\rangle \underset{\vec{k}_1+\vec{k}_2+\vec{k}_3=0}{=} -2\lambda\int_{-\infty}^{t_f} dt\, a(t)F_{k_1}(t,t_f)F_{k_2}(t,t_f)G_{k_3}(t_f,t) \quad \text{on-shell}$$

$$\text{off-shell} \quad +\frac{1}{4}\lambda\int_{-\infty}^{t_f} dta(t)G_{k_1}(t_f,t)G_{k_2}(t_f,t)G_{k_3}(t_f,t) + k-perm. + O(\lambda^2)$$

$=$  $+$ …

- When including leading **terms of general relativity** [7], we need to look at

$$[g_1(t)\dot{\mathcal{R}}^3] + g_2(t)\mathcal{R}\dot{\mathcal{R}}^2 + g_3(t)\mathcal{R}(\partial\mathcal{R})^2 + g_4(t)\dot{\mathcal{R}}\partial\mathcal{R}\partial\chi \subset \mathcal{H}^{(3)}$$

- Analytic expressions computed for the first time in this formalism, e.g.

$$\begin{cases} \langle \hat{\mathcal{R}}_{\mathbf{k}_1}\hat{\mathcal{R}}_{\mathbf{k}_2}\hat{\mathcal{R}}_{\mathbf{k}_3}|_{-\infty}^{\eta_f}\rangle^{\text{on-shell}}_{\mathcal{R}\dot{\mathcal{R}}^2} \propto \dfrac{e_1^2\left(3\eta_f^2e_2-2\right)e_3 + e_2\left(1-\eta_f^2e_2\right)e_3 + e_1e_2\left(-e_3^2\eta_f^4 - e_2^2\eta_f^2 + e_2\right)}{2e_1^2e_3^3} \\ \langle \hat{\mathcal{R}}_{\mathbf{k}_1}\hat{\mathcal{R}}_{\mathbf{k}_2}\hat{\mathcal{R}}_{\mathbf{k}_3}|_{-\infty}^{\eta_f}\rangle^{\text{off-shell}}_{\mathcal{R}\dot{\mathcal{R}}^2} \propto \dfrac{\left(e_1^2-2e_2\right)\left(\eta_f^2\left(8e_2+e_1\left(e_1\left(\eta_f^2\left(e_1^2-4e_2\right)-4\right)+8\eta_f^2e_3\right)\right)-8\right)}{2e_1^2\left(e_1^3-4e_1e_2+8e_3\right)^2} \end{cases}$$

where $\quad e_1 = k_1+k_2+k_3,\ e_2 = k_1k_2+k_2k_3+k_1k_3,\ e_3 = k_1k_2k_3$

- None of the studied interaction show that we can neglect quantum dynamics.
- **Some scales are special**:
  - **Folded** limit ($k_3 \simeq k_1+k_2$): off/on-shell integrals contribute equally ()
  - **Squeezed** limit ($k_3 \ll k_1, k_2$): on-shell dominates
- On-shell functions have **physical poles** as opposed to off-shell **virtual ones.**

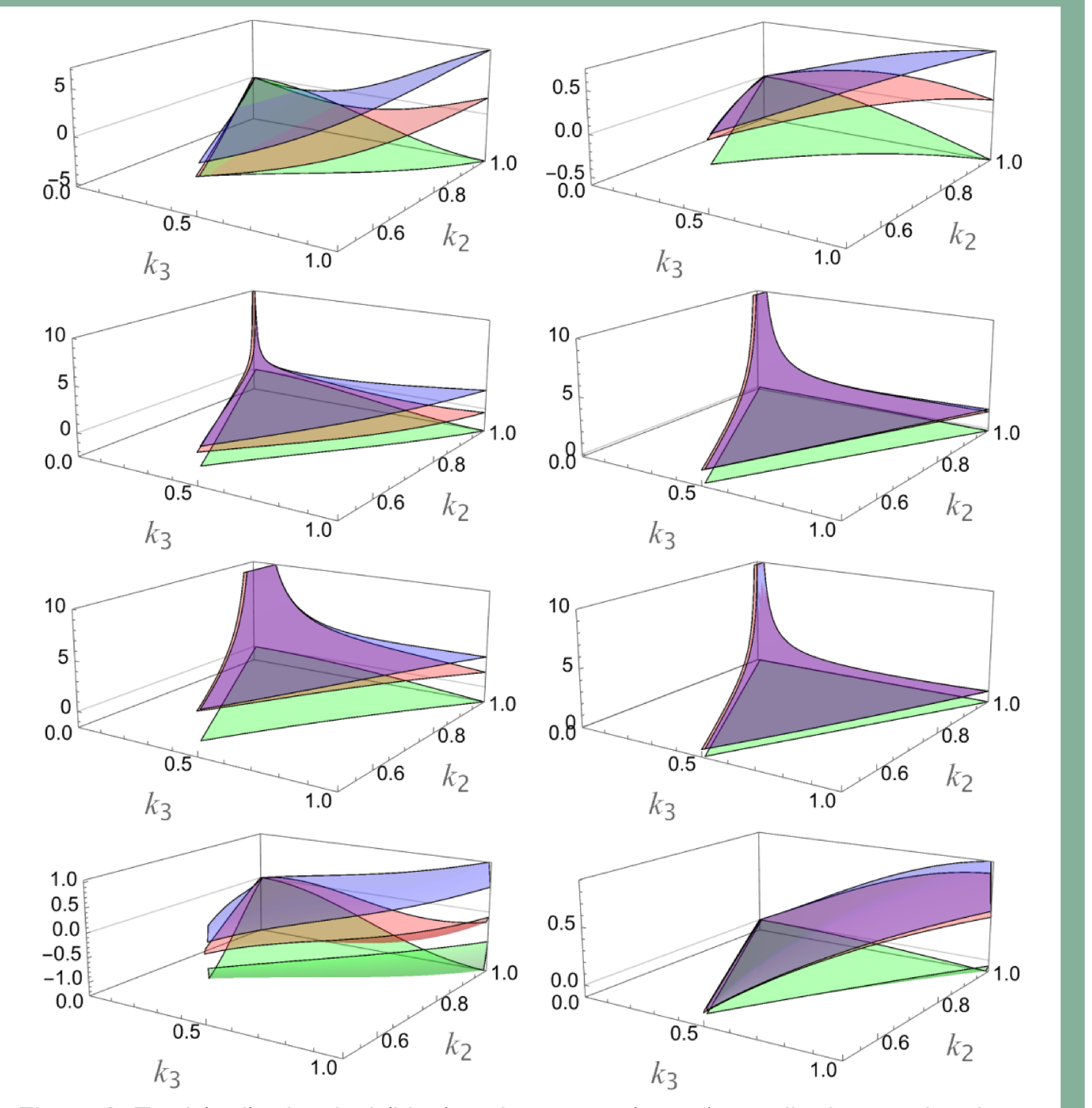


Figure 2: Total (red), classical (blue) and quantum (green) contributions to the shape function $e_3^2(\hat{\mathcal{R}}^3)$ of late integrals from $\mathcal{N}_0 = -1$ (left, equivalent to $\eta_0 \simeq -2.72$) or $\mathcal{N}_0 = -0.25$ (right, equivalent to $\eta_0 \simeq -1.28$) to $\mathcal{N}_f = 4.6$ (equivalent to $\eta_f \simeq -0.01$) for our 4 interactions in quasi-dS (top to bottom). k_1 is set to 1 and is the last mode to cross the horizon ($k_{2,3} \le k_1$), time at which we set the time origin ($k_1 = aH$ at $\eta = -1$ or $\mathcal{N} = 0$ equivalently). Plotting threshold is set at an absolute value of 10.

Timing semi-classicality

- **Starting** previous integrals (and equivalently simulations) **at a well-chosen time t_0** can make a difference for on vs off-shellness.
- We defined and studied the **quantum interactivity**

$$\mathcal{QI}(t_0,t_f,\{k_j\}) = \frac{|\langle \hat{\mathcal{R}}_{\mathbf{k}_1}\hat{\mathcal{R}}_{\mathbf{k}_2}\hat{\mathcal{R}}_{\mathbf{k}_3}|_{t_0}^{t_f}\rangle^{\text{off-shell}}|}{|\langle \hat{\mathcal{R}}_{\mathbf{k}_1}\hat{\mathcal{R}}_{\mathbf{k}_2}\hat{\mathcal{R}}_{\mathbf{k}_3}|_{t_0}^{t_f}\rangle^{\text{off-shell}}| + |\langle \hat{\mathcal{R}}_{\mathbf{k}_1}\hat{\mathcal{R}}_{\mathbf{k}_2}\hat{\mathcal{R}}_{\mathbf{k}_3}|_{t_0}^{t_f}\rangle^{\text{on-shell}}|}$$

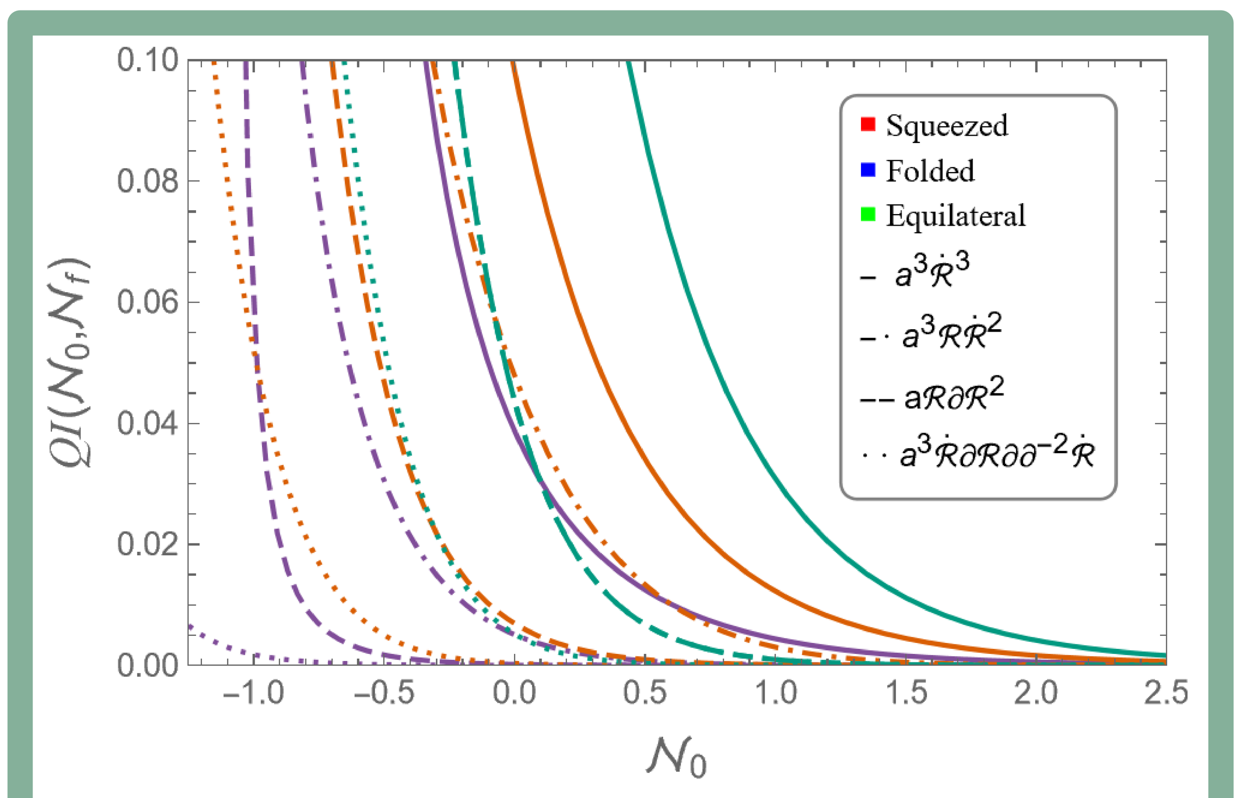


Figure 3: Bispectral quantum interactivity of our four interactions for different scales as a function of the initial time truncation. k_1 is set to 1 and is the last mode to cross the horizon ($k_{2,3} \le k_1$), time at which we set the time origin ($k_1 = aH$ at $\eta = -1$ or $\mathcal{N} = 0$ equivalently).

- The **classicality time** of the bispectrum depends on:
  - **Bispectral precision** (giving upper bound on $\mathcal{QI}$)
  - **interactions**: ~more time derivatives → later
  - **scales**: squeezed is classical early, then comes folded and equilateral.

Take-aways

- ✓ On/off-shell dynamics have **special signatures**
- ✓ On/off-shell dynamics are **comparable** sufficiently **deep in the horizon**
- ✓ **Classicality** of the 3pt function can be **reached much before** what the **commutators** say, depending on the scales and interactions.
- ✓ Towards **a validity criterion for** classical **simulations** of the early universe.

References:

[1] D. Green et al. 2020, PRL 124 251302
[2] K. Clough et al. 2017, JCAP 09 025
[3] A. Caravano et al. 2025, PRD 111 063518
[4] Y. Launay et al. 2025, 2502.06783
[5] J. Martin et al. 2016, PRD 93 023505
[6] L.V. Keldysh 1964, ZETF 47 1515
[7] J. Maldacena 2003, JHEP 05 013.

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← **Related publication**: Yoann L. Launay et al JCAP05(2025)071.

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